**MODERN COLLEGE OF ARTS,SCI. & COMM. PUNE-05.**

**DEPARTMENT OF STATISTICS.**

**ST- 28 M.Sc.( I )**

**EXPT.NO. 09**

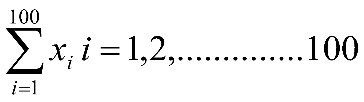
**Application and verification of central limit Theorem**

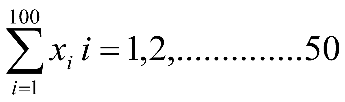
Q.1  Let X→ B (n, θ) use the central limit theorem  to find n such that,

Prob [X> n/2] ≥ 1- α

If α = 0.1 and θ= 0.45 calculate n

Satisfied prob  [X> n/2] ≥ 0.9

Q.2  Let X1, X2 ……X100 be a random sample from Poisson (λ) where λ= 3.  S=. Find prob(S≥300) using central limit theorem and compare with exact probability.

Q3. Let X1, X2 ……X50 be a random sample from Bernoulli(=0.45). S=.

(i) find prob(S≥30) using central limit theorem and compare with exact probability.

(ii) find prob(S≥10) using central limit theorem and compare with exact probability.

THEORY

Central Limit Theorem (CLT)

If Xi (i = 1,2,….n) be independently distributed random variables such that E(Xi) = μi and Var(Xi) = σi2 then as n tends to infinity the distribution of the sum of these random variables, namely Sn = X1 + X2 +….+Xn tends to the normal distribution with mean μ and variance σ2 where

and

Procedure

* Draw a random sample of size n from given distribution.
* Calculate S where S = X1 + X2 +….+X100 (n=100).
* Repeat above two steps 1000 times. Now we have S vector of size 1000.
* Calculate mean and variance of S.
* Convert this S into standard normal variate i.e. (use CLT)
* If sample size n>30 then we can use CLT.
* Then find out required probability using R command.

Sample :

|  |
| --- |
| Sum of Xi |
| S1 |
| S2 |
| .. |
| .. |
| S1000 |

|  |
| --- |
| 1 |
| 2 |
| .. |
| . |
| 1000 |

As n tends to infinity S follows N()

~ N(0,1)

* To calculate P(S)

P( = P(Z>…)

= 1 – pnorm(Z,0,1) gives approximate probability.

* To calculate exact probability P(S300)

P(S300) = 1 – ppois(299,300) gives exact probability.

Q.1  Let X→ B (n, θ) use the central limit theorem  to find n such that,

Prob [X> n/2] ≥ 1- α

          If α = 0.1 and θ= 0.45 calculate n

Satisfied prob  [X> n/2] ≥ 0.9

Prob [X> n/2] ≥ 1- α

Prob [X> n/2] ≥ 0.9

Prob [> ] ≥ 0.9

Since X ~ B (n,

Thus, = and

Therefore,

Prob [> ] ≥ 0.9

Prob [Z> ] ≥ 0.9 …..(1)

Here Z follows standard normal distribution Z~N(0,1)

Also

> qnorm(0.90,0,1)

[1] 1.281552

#Thus Prob [Z>1.281552] ≥ 0.9 …..(2)

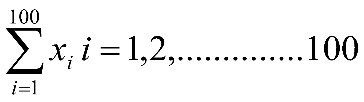
Comparing (1) and (2) we get

1.281552

1.281552 \*

0.637564069

Thus n = 163

Q.2  Let X1, X2 ……X100 be a random sample from Poisson (λ) where λ= 3.  S=. Find prob(S300) using central limit theorem and compare with exact probability

> n=1000

> s=rep(0,1000)

> for(i in 1:1000)

+ {

+ x = rpois(100,3)

+ s[i] = sum(x)

+ }

> s

[1] 312 312 285 267 330 284 294 314 271 287 295 293 317 295 316 322 304 315

[19] 311 289 284 301 309 314 276 300 324 278 296 291 309 325 300 279 313 294

[37] 298 289 300 249 294 314 284 270 280 291 292 311 286 318 303 306 309 300

[55] 295 318 302 322 306 309 280 277 283 291 282 306 308 280 314 309 312 292

[73] 291 286 325 298 338 295 302 311 297 320 300 293 303 293 298 327 317 309

[91] 289 303 281 328 332 305 281 302 306 309 316 301 292 303 315 326 310 289

[109] 319 321 317 269 276 279 285 282 287 275 297 309 303 296 295 309 257 298

[127] 284 295 301 321 337 303 325 327 303 296 277 322 274 344 306 302 334 288

[145] 313 289 290 322 272 307 284 306 300 302 279 277 281 292 288 306 276 280

[163] 298 301 294 280 310 294 288 301 310 306 323 308 308 293 298 297 279 300

[181] 284 281 283 282 326 290 288 321 301 296 293 287 309 316 305 316 257 289

[199] 269 300 300 307 334 311 339 317 312 297 290 287 311 313 309 308 318 306

[217] 327 317 290 290 298 313 275 287 288 309 297 299 297 303 311 290 314 290

[235] 286 345 302 306 289 291 310 334 316 319 309 268 307 278 273 302 289 303

[253] 285 304 317 307 290 326 295 303 297 313 281 290 303 313 326 291 300 311

[271] 302 296 311 286 276 302 298 286 337 324 303 286 292 307 298 283 297 283

[289] 306 274 288 276 323 316 304 307 283 291 297 269 298 299 325 303 320 280

[307] 307 295 296 313 275 262 307 318 281 311 320 286 282 321 316 290 296 287

[325] 314 280 308 309 298 283 321 304 270 303 275 302 278 330 323 300 317 280

[343] 278 303 285 286 283 323 299 290 308 314 298 254 288 288 285 316 290 268

[361] 308 275 316 322 313 295 290 268 285 310 295 315 288 311 332 292 278 328

[379] 291 291 317 296 287 305 288 278 303 325 325 290 346 294 323 293 313 310

[397] 308 323 276 304 305 319 311 288 279 301 317 291 283 283 293 283 312 312

[415] 283 283 302 292 279 319 304 286 299 306 309 310 270 310 302 314 294 306

[433] 300 332 273 237 295 291 306 313 283 322 308 307 318 311 305 289 318 321

[451] 301 323 293 313 272 289 270 306 311 304 282 295 312 301 279 340 318 322

[469] 314 307 279 314 273 314 314 336 297 292 321 284 296 324 300 314 315 292

[487] 314 308 274 295 321 267 271 312 306 297 292 303 301 299 318 316 299 303

[505] 236 311 310 296 328 322 297 279 298 279 303 297 309 289 304 312 292 323

[523] 313 310 284 279 310 299 294 296 325 255 296 279 336 288 293 312 309 314

[541] 274 309 287 291 311 260 297 301 268 311 301 301 288 286 273 310 287 313

[559] 280 284 282 319 317 287 262 302 300 310 280 323 273 284 271 304 321 311

[577] 314 275 314 322 298 279 314 319 282 315 272 285 300 301 305 312 280 279

[595] 307 302 326 315 272 280 292 297 291 307 303 305 310 311 310 306 310 272

[613] 267 266 308 314 291 295 320 316 332 290 353 305 285 345 308 331 299 290

[631] 274 306 309 313 279 313 298 280 277 301 325 275 321 321 308 335 284 297

[649] 274 328 266 283 328 317 287 310 310 298 325 304 308 294 362 299 297 306

[667] 331 311 295 305 282 278 297 289 292 333 298 275 297 313 304 329 294 302

[685] 323 273 282 303 316 281 295 313 282 301 272 283 332 292 314 306 301 285

[703] 274 290 309 298 296 302 311 286 281 290 280 303 278 294 306 320 325 298

[721] 283 288 315 337 307 302 309 308 284 294 297 298 306 305 313 276 291 301

[739] 305 306 298 299 292 309 338 320 247 257 318 334 305 271 314 323 310 312

[757] 308 280 297 284 311 286 301 260 309 294 295 304 323 293 341 302 277 305

[775] 264 270 278 337 290 303 302 303 303 278 312 276 300 309 271 312 338 312

[793] 289 308 303 320 309 295 299 282 300 298 286 294 316 328 298 292 292 301

[811] 302 299 297 299 299 301 298 320 297 307 284 308 331 290 297 315 309 336

[829] 273 318 300 323 294 311 322 284 317 307 284 270 301 280 314 279 267 288

[847] 294 320 281 290 309 284 295 307 304 330 292 306 287 290 325 312 277 283

[865] 317 289 307 323 333 297 282 280 273 330 315 323 291 287 283 317 306 280

[883] 289 288 300 324 291 296 285 303 311 278 286 289 308 294 324 299 299 297

[901] 289 329 305 285 305 285 306 288 348 295 277 283 267 303 282 318 290 272

[919] 295 305 286 295 303 293 317 303 311 310 321 304 290 297 286 309 288 283

[937] 305 307 308 288 307 287 283 335 286 306 289 273 309 313 294 312 230 306

[955] 318 287 298 300 304 306 291 299 315 300 286 272 300 312 318 282 306 292

[973] 316 341 310 284 313 324 316 299 318 311 326 295 307 271 290 298 306 320

[991] 341 292 274 291 306 314 273 297 271 276

> mean = mean(s)

> mean

[1] 299.824

> var = (n/(n-1))\*var(s)

> var

[1] 298.622

> z=(300-mean)/sqrt(var)

> #required prob using CLT P(S>=300)=1-P(S<300)

> p1 = 1-pnorm(z,0,1)

> p1

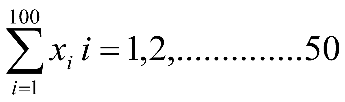
[1] 0.4959369

> #exact probability P(S>=300)=1-P(S<300)

> p2 = 1-ppois(299,300)

> p2

[1] 0.5076778

Q3. Let X1, X2 ……X50 be a random sample from Bernoulli(=0.45). S=.

(i) find prob(S30) using central limit theorem and compare with exact probability.

> n=1000

> s=rep(0,1000)

> for(i in 1:1000)

+ {

+ x = rbinom(50,1,0.45)

+ s[i] = sum(x)

+ }

> s

[1] 19 26 25 20 23 23 24 15 25 30 23 22 21 26 20 23 35 21 21 18 22 22 26 25

[25] 19 23 28 22 29 25 21 22 21 22 25 27 26 26 25 23 17 24 22 24 18 28 26 21

[49] 25 18 21 25 20 21 19 25 19 21 25 21 24 18 18 25 25 19 26 26 21 23 20 21

[73] 29 27 20 18 24 23 26 25 26 24 20 28 27 22 20 24 20 27 20 17 15 29 16 24

[97] 18 24 23 20 25 18 24 24 23 18 23 26 24 26 25 21 21 23 18 25 23 25 26 20

[121] 22 24 22 19 25 19 21 20 21 22 28 20 30 19 21 20 22 25 21 26 25 21 23 25

[145] 22 28 25 16 23 26 19 21 22 19 23 22 24 24 13 19 25 22 18 17 23 20 20 20

[169] 22 20 17 18 19 25 20 23 18 20 15 19 18 23 28 24 21 25 24 26 28 23 21 26

[193] 23 20 23 19 19 24 23 25 22 25 22 27 23 20 22 24 20 25 21 25 27 25 21 21

[217] 27 23 22 27 28 24 22 31 27 20 22 21 19 28 24 20 16 24 25 20 23 24 18 24

[241] 22 19 19 16 20 21 26 19 34 23 23 19 26 26 24 22 23 22 29 20 24 25 24 25

[265] 24 26 19 27 19 22 23 21 24 19 26 23 17 22 23 22 28 19 23 25 23 26 24 18

[289] 18 21 24 20 21 30 25 25 27 15 22 22 14 20 29 24 23 23 23 25 25 23 26 14

[313] 19 22 17 26 23 27 24 25 29 20 25 24 28 20 21 24 19 28 20 23 21 28 18 26

[337] 19 21 14 27 23 24 18 24 25 14 28 24 17 26 21 26 19 21 30 27 24 18 23 22

[361] 17 20 26 20 32 16 18 23 23 20 21 24 19 24 21 20 25 25 21 29 22 27 22 19

[385] 20 17 21 26 19 25 21 21 28 16 19 18 30 26 25 26 26 18 21 22 26 22 25 26

[409] 24 22 25 25 22 31 26 19 22 23 27 20 24 24 26 26 21 22 22 25 24 19 24 25

[433] 28 24 21 31 21 22 18 22 16 21 19 24 21 27 20 16 25 23 23 25 23 20 14 16

[457] 23 18 19 22 21 16 24 19 27 14 26 26 25 22 16 22 22 17 21 23 21 25 22 21

[481] 22 22 21 23 21 20 31 19 23 23 19 22 17 22 21 25 19 25 13 24 19 22 17 29

[505] 18 21 24 20 25 20 19 23 21 18 25 31 30 28 21 23 20 24 23 27 21 28 20 24

[529] 19 23 17 17 24 24 24 26 29 24 19 18 22 23 18 23 25 20 24 26 24 26 25 28

[553] 27 24 19 25 22 25 16 19 20 18 20 28 18 19 23 26 25 23 25 25 27 13 26 25

[577] 21 29 28 20 23 23 22 19 23 27 25 21 29 23 21 22 20 17 17 13 23 20 29 20

[601] 19 28 15 22 25 23 25 28 20 28 25 18 25 26 20 26 25 19 24 19 15 27 18 19

[625] 24 23 23 24 20 17 24 18 23 20 23 24 16 24 19 20 19 23 31 25 26 25 23 22

[649] 21 31 28 23 24 29 22 26 25 29 18 25 21 28 22 20 26 23 17 27 23 18 27 26

[673] 14 22 17 15 19 28 21 20 29 20 21 21 21 22 19 22 18 20 28 18 23 25 21 16

[697] 21 26 21 28 25 28 23 23 29 19 24 23 26 27 18 19 21 23 30 19 22 19 28 26

[721] 25 17 26 22 25 25 23 26 24 24 17 18 17 22 17 27 22 28 21 28 20 18 22 21

[745] 18 21 22 24 19 21 22 23 20 17 21 16 20 19 26 23 22 22 26 19 16 17 16 27

[769] 25 21 22 21 24 24 20 27 20 26 30 17 15 26 28 22 20 23 21 20 22 23 21 18

[793] 21 24 23 27 24 20 30 18 28 19 24 20 25 20 22 22 29 25 22 23 27 30 19 29

[817] 21 24 24 24 23 30 23 25 22 30 20 24 27 20 29 27 20 13 17 26 22 18 23 21

[841] 17 23 24 15 21 14 25 22 15 24 20 23 21 23 21 25 22 21 17 22 23 20 26 23

[865] 17 20 24 14 24 30 23 20 24 25 21 17 29 20 22 19 22 22 20 27 25 22 23 20

[889] 21 24 16 26 21 23 21 28 25 20 22 20 25 23 24 18 25 24 23 25 22 24 30 24

[913] 27 26 19 18 18 25 26 19 23 26 23 23 18 17 17 26 18 20 19 22 23 26 20 22

[937] 20 19 21 24 23 18 23 20 18 22 22 15 22 18 22 27 26 24 20 17 30 17 22 27

[961] 25 24 23 21 17 19 21 22 20 26 25 17 26 23 24 30 23 22 21 16 24 26 21 19

[985] 26 25 25 21 25 26 21 24 25 17 18 19 23 25 15 29

> mean = mean(s)

> mean

[1] 22.458

> var = (n/(n-1))\*var(s)

> var

[1] 12.8399

> z=(30-mean)/sqrt(var)

> #required prob using CLT P(S>=30)=1-P(S<30)

> p1 = 1-pnorm(z,0,1)

> p1

[1] 0.01765545

> #exact probability P(S>=30)=1-P(S<30)

> p2 = 1-pbinom(29,50,0.45)

> p2

[1] 0.02353582

(ii) find prob(S10) using central limit theorem and compare with exact probability.

> n=1000

> s=rep(0,1000)

> for(i in 1:1000)

+ {

+ x = rbinom(50,1,0.45)

+ s[i] = sum(x)

+ }

> s

[1] 25 23 22 26 25 22 17 27 21 22 23 23 29 25 19 20 19 18 28 27 28 25 26 24

[25] 25 23 24 25 21 24 27 17 28 20 20 30 22 28 19 15 20 21 25 22 27 22 19 27

[49] 30 22 18 23 18 22 19 24 19 18 25 25 23 28 18 21 16 26 26 23 24 18 23 20

[73] 19 31 27 26 23 21 29 18 17 19 18 18 25 24 20 28 21 24 23 25 27 22 19 21

[97] 24 25 19 27 17 22 20 26 25 19 23 26 20 23 23 24 20 24 30 20 16 22 22 26

[121] 18 22 22 24 25 22 25 22 20 20 21 25 24 22 26 22 24 24 25 25 18 21 18 25

[145] 26 28 21 21 26 21 21 21 20 23 19 24 25 20 21 23 25 26 23 26 22 15 24 23

[169] 24 25 19 23 22 21 20 20 27 27 20 13 21 25 21 22 23 17 34 25 16 22 20 22

[193] 23 25 24 22 26 17 23 24 19 27 25 18 21 32 27 23 22 21 24 17 23 26 18 21

[217] 19 22 25 17 20 25 30 22 25 23 17 20 21 19 20 24 24 24 25 20 22 23 21 29

[241] 22 22 23 19 25 25 20 28 29 18 30 27 16 23 26 18 22 22 15 26 18 24 25 21

[265] 21 20 17 18 22 23 22 22 17 14 27 25 18 19 15 24 21 24 18 26 23 22 27 25

[289] 21 20 22 22 25 22 20 19 25 19 23 24 21 17 30 25 26 25 28 27 23 20 22 21

[313] 29 23 25 16 25 22 21 21 30 23 29 21 27 30 24 21 19 24 24 24 24 22 22 22

[337] 32 26 16 21 27 21 17 24 25 29 18 24 21 28 23 19 21 16 28 23 21 17 23 25

[361] 19 21 18 26 18 25 22 21 23 25 21 26 23 27 16 23 22 28 18 23 26 20 21 25

[385] 21 20 23 20 27 17 26 28 24 28 27 19 22 23 23 26 29 21 22 21 22 24 25 23

[409] 23 20 23 24 21 20 21 21 22 21 19 16 21 24 20 26 22 19 24 26 24 22 17 21

[433] 26 20 25 18 29 21 23 25 25 23 29 24 24 23 17 23 21 25 24 21 18 24 22 20

[457] 24 21 26 15 21 24 21 16 27 28 26 24 21 16 23 26 28 18 21 22 22 19 19 21

[481] 18 16 18 21 24 22 25 25 25 16 27 23 23 17 24 21 25 21 26 22 23 18 30 25

[505] 27 18 23 22 20 22 25 26 27 28 20 19 25 16 23 26 20 27 16 21 23 17 19 22

[529] 25 19 19 22 18 22 22 17 27 14 23 19 24 23 21 22 18 13 19 16 26 24 21 26

[553] 19 19 25 26 31 23 25 29 24 22 24 23 20 22 23 19 23 19 22 26 24 14 22 23

[577] 24 18 20 26 27 21 21 21 24 28 22 28 22 30 24 23 27 21 21 23 24 23 26 23

[601] 31 23 24 21 23 24 24 19 21 22 20 18 24 26 19 25 22 20 24 19 21 17 23 25

[625] 21 27 17 18 21 27 17 27 19 31 19 20 24 24 18 23 22 22 24 15 27 22 30 20

[649] 28 19 25 27 25 26 26 21 16 19 22 20 17 18 19 17 26 23 19 24 26 22 18 25

[673] 22 22 18 20 15 26 23 27 22 18 23 20 24 23 21 20 21 21 20 21 22 19 26 21

[697] 21 22 24 20 25 28 17 23 20 20 30 25 24 25 27 18 21 27 17 23 25 22 19 25

[721] 14 17 23 22 17 23 24 23 28 19 16 22 22 26 26 19 26 25 27 21 10 24 32 21

[745] 21 27 23 31 22 17 21 20 22 24 20 17 21 21 18 22 26 17 26 22 21 18 23 22

[769] 23 19 20 23 20 24 22 25 22 24 26 20 16 24 14 20 21 21 22 19 23 22 25 23

[793] 22 21 25 19 14 25 26 30 23 20 19 23 27 24 18 17 26 15 22 21 26 22 18 27

[817] 22 16 24 24 29 23 23 25 25 25 21 25 20 27 25 23 22 21 19 21 13 29 25 27

[841] 20 19 19 27 20 27 25 26 20 19 24 24 27 23 17 21 19 24 18 20 25 21 31 12

[865] 22 25 22 21 21 22 20 19 23 26 22 24 25 18 20 25 17 17 27 24 22 23 17 27

[889] 18 20 23 21 26 19 22 25 25 26 22 24 23 22 21 18 18 24 19 22 27 28 21 29

[913] 14 24 15 30 24 15 29 21 25 20 17 25 30 23 24 25 24 21 13 18 18 12 23 24

[937] 30 23 24 20 22 27 24 25 25 25 26 26 20 20 25 21 28 24 24 23 23 20 19 22

[961] 19 24 23 19 20 17 20 28 28 27 25 19 15 21 27 25 21 25 23 23 26 23 22 20

[985] 25 22 20 28 26 23 21 20 27 22 30 15 27 18 20 27

> mean = mean(s)

> mean

[1] 22.422

> var = (n/(n-1))\*var(s)

> var

[1] 12.65121

> z=(10-mean)/sqrt(var)

> #required prob using CLT P(S>=10)=1-P(S<10)

> p1 = 1-pnorm(z,0,1)

> p1

[1] 0.9997607

> #exact probability P(S>=10)=1-P(S<10)

> p2 = 1-pbinom(10,50,0.45)

> p2

[1] 0.9997987